Theory of resonant and stimulated excitation of magnetic-moment fields in wave-plasma interactions

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Theories of two nonlinear processes of magnetic-moment-field generation in wave-plasma interactions are presented here. These processes are (i) resonant excitation of a moment field (REMF) and (ii) stimulated excitation of a moment field (SEMF). This field generally evolves from the wave-induced bending of the direction of motion of constituents of a plasma. It is important when it has a large value, and when it grows to large values with time, because then it effectively controls the wave-induced features of a plasma. Specifically, this growing field gives rise to a strong anisotropy of the plasma in the region of the common direction of propagation of the involved waves, which leads to enhancement of synchrotron and bremmsstrahlung losses, and filamentation. The REMF is a static field of resonance when the beat frequency of two waves equals the frequency of another wave, all propagating in the same direction. The three possible cases of such interaction, involving waves of only high frequencies, with unmagnetized plasmas, for which the REMF formula has been calculated, are (a) two transverse waves and one longitudinal wave, (b) two longitudinal waves and one transverse wave, and (c) three transverse waves. The SEMF is a parametrically stimulated nonoscillating, exponentially temporally growing field of stimulated Brillouin scattering from a signal Alfven wave, a pump Alfven wave, and a signal sound wave. A second simultaneous resonance occurs only for weak nonlinearity and finite electrical conductivity, when the signal Alfven wave frequency equals the parametric frequency shift. This, being a slow process of transfer of plasma kinetic energy to field energy, can be a strong candidate for evolution of the field in plasma configurations of outer space.

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I. INTRODUCTION

The magnetic moment $\boldsymbol{\mu}$ of a single charge q, in the cgs Gaussian system of units, is given by [1]

$$\boldsymbol{\mu}=\frac{1}{2c}(\mathbf{r}\times\mathbf{j}) , \tag{1.1}$$

where $\mathbf{r}$ is its position vector, $\mathbf{j}(=q\dot{\mathbf{r}})$ is its current, and $\dot{\mathbf{r}}$ is its velocity. For several species of charges, in a plasma, the expression for the induced magnetization from magnetic moment [2,3] per unit volume is

$$\mathbf{H}^{\rm in}=4\pi\sum_p \boldsymbol{\mu}_p=4\pi\boldsymbol{\mu}=\frac{4\pi}{2c}\sum_p(\mathbf{r}_p\times\mathbf{j}_p)=\frac{4\pi}{2c}\sum_p[\mathbf{r}_p\times(N_pq_p\dot{\mathbf{r}}_p)] , \tag{1.2}$$

where q_p is the charge per particle, N_p is the number density, $\dot{\mathbf{r}}_p$ is the macroscopic velocity at $\mathbf{r}_p$, and $\boldsymbol{\mu}_p$ is the magnetic moment per unit volume of the pth species of charges.

If, in a two-component plasma, the ions provide only a static background of positive charges, for maintaining the macroscopic charge neutralization, then (1.2) reduces to

$$\mathbf{H}^{\rm in}=-\frac{4\pi Ne}{2c}(\mathbf{r}_e\times\dot{\mathbf{r}}_e) . \tag{1.3}$$

In a multispecies plasma we can write

$$\mathbf{r}_p=\mathbf{R}+\boldsymbol{\xi}_p , \tag{1.4}$$

where $\mathbf{R}$ is the position vector of a current point before application of the waves, and $\boldsymbol{\xi}_p$ is the field-induced displacement of the pth species of its charged constituents from the point $\mathbf{R}$. Since $\mathbf{R}$ is not a wave-induced displacement of a specific species of charge unlike $\mathbf{r}$ and $\boldsymbol{\xi}$, it should not have the subscript p for specification. Then, if we denote by $\langle x\rangle$ the nonoscillating part (or the zeroth harmonic) of x, Eq. (1.2) gives

$$\langle\mathbf{H}^{\rm in}\rangle=4\pi\langle\boldsymbol{\mu}\rangle=\frac{4\pi}{2c}\sum_p\langle\{\boldsymbol{\xi}_p\times(N_pq_p\dot{\boldsymbol{\xi}}_p)\}\rangle . \tag{1.5}$$

For a single, electrically conducting fluid, the induced nonoscillating field $\mathbf{H}^{\rm in}$ obtains from the formula

$$\langle\mathbf{H}^{\rm in}\rangle=4\pi\langle\boldsymbol{\mu}\rangle=\frac{4\pi}{2c}\langle(\boldsymbol{\xi}\times\mathbf{j})\rangle , \tag{1.6}$$

where $\boldsymbol{\xi}$ is the wave-induced displacement of the fluid element at a point, and the current $\mathbf{j}$ is given in terms of the magnetic induction field $\mathbf{H}$, by the Maxwell equation

$$\nabla\times\mathbf{H}=\frac{4\pi}{c}\mathbf{j} , \tag{1.7}$$

which is without the displacement current, and is used in the magnetohydrodynamics (MHD) approximation.

Several theoretical problems of temporal evolution of the magnetic-moment field, due to the bending of direction of material motion in presence of wave fields, can be formulated and studied, because plasmas generate and support many categories of waves and oscillations. This is an effective process of transfer of kinetic energy to the magnetization energy. Since this field is proportional to the vector product of a displacement and a current, both induced by waves, the contribution due to quadratic nonlinearities arises from the sum of two cross products, one of which is that of a first-order displacement and a second-order current, and the other is that of a first-order current and a second-order displacement. The relevant second-order quantities are those of mixed harmonic of two of the waves, which on cross product with the field of first harmonic of the other wave, generate a magnetic field of the zeroth harmonic due to a prescribed matching of frequencies of first harmonic of the three waves. The wave-induced displacement $\mathbf{r}$ and current $\mathbf{j}$, satisfying the nonlinear field equations, are expressed as a sum of all harmonics, including some mixed harmonic as well. Then the zeroth harmonic of $\mathbf{H}^{\text{in}}$, proportional to $(\mathbf{r}\times\mathbf{j})$, obtains in many cases of combination of wave fields of different categories. This magnetization from some three-wave interactions is evidently an outcome of either Compton scattering, or stimulated Brillouin scattering (SBS), or stimulated Raman scattering (SRS) in plasmas, depending on the nature of the involved waves.

This moment field was originally found in the interaction of a circularly polarized wave with crystals in the 1960s [4,5] and with plasmas in the 1970s [6]. It was then called the inverse Faraday effect (IFE) because, for circularly polarized waves, it is essentially the inverse of the Faraday rotation effect.

We have considered here the theories of generation of (I) a static magnetic field from a matching of frequencies of first harmonic of three high-frequency waves in an unmagnetized electron plasma, and (II) parametrically temporally exponentially growing magnetization, from a matching of frequencies of a pump Alfven wave, a signal Alfven wave, and a signal sound wave in an electrically conducting medium. Evidently, case I is identified as the resonant excitation of a moment field (REMF) because of resonance of a mixed (nonlinearly) harmonic wave with the first harmonic of another wave. Case II, on the other hand, is identified as the stimulated excitation of the moment field (SEMF) because it is the nonoscillating field from a wave field which grows parametrically due to stimulated scattering processes [7,8]. The region where the matching condition for the wave frequencies is satisfied is small compared to the largest of the wavelengths of the involved waves. Such common regions might exist where the waves cross each other, or where these are reflected. Works on magnetization from other causes have been briefly covered in Sec. V A.

Case I. Consider the magnetic moment due to plasma motion induced by resonance of three waves, when the sum frequency of two of the waves equals the frequency of the other wave. Then one term of this moment field becomes a constant. This term is important if it has a sizable value for waves of occurrence in space and laboratory plasmas. The three possible problems of such interaction of waves of only high frequency with a plasma are those of (a) two transverse waves and one longitudinal wave, (b) two longitudinal waves and one transverse wave, and (c) three transverse waves. Since the longitudinal waves are of high frequencies, these are assumed to be only the electron acoustic waves.

Case II. The evolution of an exponentially temporally growing magnetic field dependent on the pump power, when it exceeds a threshold value, is predicted in the three-wave resonance of SBS of an Alfven wave (wave 2) and a sound wave (wave 1) by an Alfven wave (wave 3) in a finitely electrically conducting MHD fluid, when

$$\omega_1+\omega_2=\omega_3 \; . \tag{1.8}$$

This SEMF is further enhanced when the signal Alfven wave frequency ω_2 equals the parametric frequency shift ω [which is the real part of the complex frequency $\bar{\omega}$ of the relation (3.41)],

$$\omega_2=\omega \; . \tag{1.9}$$

This double resonance from (1.8) and (1.9) is shown in Sec. IV B to appear only for weak nonlinearity and finite electrical conductivity. For infinite conductivity both energy and momentum of the system remain conserved, and this double resonance effect is not possible.

II. RESONANT EXCITATION OF A MOMENT FIELD

For high-frequency electromagnetic (EM) waves, and weak ambient magnetic fields, the plasma can be assumed to be unmagnetized, only the electron current is important, and the ion current is not a dominating factor for wave fields of moderate or weak intensity. Since the higher harmonic fields are weaker than the corresponding first harmonic fields, the main contribution to REMF appears from a matching of frequencies of the first harmonic of the waves. So, the REMF is proportional to products, taken three at a time, of the wave amplitudes.

When two of the three waves are transverse waves [namely, in subcases (a) and (c)], the REMF exists only in a lateral direction with respect to the common direction of propagation of the three waves. So it causes anomalous diffusion of plasma in the presence of wave fields. All these waves have a cutoff frequency, so the prescribed frequency matching generates a mismatch between the wave numbers, and a corresponding sinusoidal space variation of the magnetization along the direction of propagation of the waves. This gives rise to a longitudinal gradient in subcase (a) and a transverse gradient in subcases (b) and (c). The distance between the successive regions of maximum and minimum of magnetization, due to the gradient, is of the order of C_s/ω_p in subcase (a), C_s/ω in (b), and c/ω in (c), where ω_p is the Langmuir frequency of the plasma, C_s is the electron sound speed, c is the vacuum speed of light, and ω is the frequency of the transverse wave. Moreover, a charge-dependent drift and the consequent electric current of instability are possible. And, since a static magnetization enhances diffusion of plasma in its direction, subcases (b) and (c) augment anomalous diffusion of plasma in the presence of EM waves. The longitudinal gradient of (a) gives rise to bunching of charges at the positions of maximum of the sinusoidally varying fields; in other words, a magnetic bottling of the plasma occurs with necks at these positions.

The REMF, in unmagnetized plasmas, is an instantaneous effect of the time of switch on of the wave fields in the region where the frequency matching occurs so that, after a short duration, the guiding plasma field equations used for this study will no longer hold.

A. The basic equations

The equations of unmagnetized, and collisionally undamped, electron plasmas are

$$\frac{\partial N}{\partial t}+\nabla\cdot(N\mathbf{u})=0\ , \tag{2.1}$$

$$\frac{\partial \mathbf{u}}{\partial t}+\frac{e}{m}\mathbf{E}+\frac{C_s^2}{N_0}\nabla N=-(\mathbf{u}\cdot\nabla)\mathbf{u}-\frac{e}{mc}(\mathbf{u}\times\mathbf{H})\ , \tag{2.2}$$

$$\nabla\times\mathbf{E}=-\frac{1}{c}\frac{\partial \mathbf{H}}{\partial t}\ , \tag{2.3}$$

$$\nabla\times\mathbf{H}=\frac{1}{c}\frac{\partial \mathbf{E}}{\partial t}-\frac{4\pi eN}{c}\mathbf{u}\ , \tag{2.4}$$

$$\nabla\cdot\mathbf{E}=-4\pi eN\ , \tag{2.5}$$

$$\nabla\cdot\mathbf{H}=0\ . \tag{2.6}$$

Here $\mathbf{u}$ is the perturbation velocity of the electron fluid; N_0 is the electron number density in the unperturbed state, p is the electron pressure perturbation, ρ $(=mN)$ is the perturbed electron mass density, ρ_0 $(=mN_0)$ is the electron mass density in the unperturbed state, C_s is the electron sound speed, and $\mathbf{E}$ and $\mathbf{H}$ are, respectively, the electric and magnetic wave fields.

The first harmonic of the ith elliptically polarized transverse wave, having the frequency $\omega_{\perp i}$, wave number $k_{\perp i}$, and electric field $\mathbf{E}_{\perp i}$ is

$$\mathbf{E}_{\perp i}=(a_{\perp i}\cos\theta_{\perp i},b_{\perp i}\sin\theta_{\perp i},0)\ , \tag{2.7}$$

where

$$\theta_{\perp i}=k_{\perp i}z-\omega_{\perp i}t \tag{2.8}$$

because all the waves are assumed to propagate parallel to the z direction. The first harmonic of the ith longitudinal wave electric field is

$$\mathbf{E}_{\parallel i}=(0,0,a_{\parallel i}\cos\theta_{\parallel i}+b_{\parallel i}\sin\theta_{\parallel i})\ , \tag{2.9}$$

with

$$\theta_{\parallel i}=k_{\parallel i}z-\omega_{\parallel i}t\ . \tag{2.10}$$

The corresponding familiar dispersion relations of the linearized approximation are

$$k_{\perp i}^2c^2=\omega_{\perp i}^2-\omega_p^2 \tag{2.11}$$

for the ith transverse wave, and

$$k_{\parallel i}^2C_s^2=\omega_{\parallel i}^2-\omega_p^2 \tag{2.12}$$

for the ith longitudinal wave of the electron acoustic type, where $C_s \ll C$, and the electron plasma frequency ω_p is given by

$$\omega_p^2=\frac{4\pi Ne^2}{m}\ . \tag{2.13}$$

The conditions for free propagation of these two types of waves are

$$\omega_{\perp i}^2>\omega_p^2,\quad \omega_{\parallel i}^2>\omega_p^2\ . \tag{2.14}$$

Solving now Eqs. (2.1)–(2.6), up to first order, for $\mathbf{H}_i$, N_i, $\mathbf{u}_{\perp i}$, $\mathbf{r}_{\perp i}$, and $\mathbf{r}_{\parallel i}$, we find that

$$\mathbf{H}_i=\frac{mc^2}{e}k_{\perp i}(-\beta_{\perp i}\sin\theta_{\perp i},\alpha_{\perp i}\cos\theta_{\perp i},0)\ , \tag{2.15}$$

$$\mathbf{u}_{\perp i}=c(\alpha_{\perp i}\sin\theta_{\perp i},-\beta_{\perp i}\cos\theta_{\perp i},0)\ , \tag{2.16}$$

$$\mathbf{r}_{\perp i}=\frac{e}{\omega_{\perp i}}(\alpha_{\perp i}\cos\theta_{\perp i},\beta_{\perp i}\sin\theta_{\perp i},0)\ , \tag{2.17}$$

$$\mathbf{u}_{\parallel i}=c\left[0,0,\frac{\omega_{\parallel i}^2}{\omega_p^2}\alpha_{\parallel i}\sin\theta_{\parallel i}-\frac{\omega_{\parallel i}^2}{\omega_p^2}\beta_{\parallel i}\cos\theta_{\parallel i}\right]\ , \tag{2.18}$$

$$\mathbf{r}_{\parallel i}=\frac{c\omega_{\parallel i}}{\omega_p^2}(0,0,\alpha_{\parallel i}\cos\theta_{\parallel i}+\beta_{\parallel i}\sin\theta_{\parallel i})\ , \tag{2.19}$$

$$N_i=\frac{1}{4\pi e}\frac{mc}{e}k_{\parallel i}\omega_{\parallel i}(\alpha_{\parallel i}\sin\theta_{\parallel i}-\beta_{\parallel i}\cos\theta_{\parallel i})\ , \tag{2.20}$$

where

$$(\alpha_{\perp i},\beta_{\perp i})=\frac{e}{mc\omega_{\perp i}}(a_{\perp i},b_{\perp i})\ , \tag{2.21}$$

$$(\alpha_{\parallel i},\beta_{\parallel i})=\frac{e}{mc\omega_{\parallel i}}(a_{\parallel i},b_{\parallel i})\ . \tag{2.22}$$

$\mathbf{r}_i$ is the displacement induced by the ith wave; its component, induced by the ith transverse wave, is $\mathbf{r}_{\perp i}$, and by the ith longitudinal wave is $\mathbf{r}_{\parallel i}$. Evidently the α's and the β's are dimensionless amplitudes of the electric field.

B. The case of two transverse and one electron acoustic waves

In this case [subcase (a)] the electric field is

$$\mathbf{E}_1=(a_{\perp 1}\cos\theta_{\perp 1}+a_{\perp 2}\cos\theta_{\perp 2},b_{\perp 1}\cos\theta_{\perp 1}+b_{\perp 2}\sin\theta_{\perp 2},a_{\parallel}\cos\theta_{\parallel}+b_{\parallel}\sin\theta_{\parallel})\ . \tag{2.23}$$

The first-order solutions of Eqs. (2.1)–(2.6) are

$$\mathbf{H}_1=\frac{mc^2}{e}(-\beta_{\perp 1}k_{\perp 1}\sin\theta_{\perp 1}-\beta_{\perp 2}k_{\perp 2}\sin\theta_{\perp 2},\alpha_{\perp 1}k_{\perp 1}\cos\theta_{\perp 1}+\alpha_{\perp 2}k_{\perp 2}\cos\theta_{\perp 2},0)\ , \tag{2.24}$$

$$N_1=\frac{mc}{4\pi e^2}k_{\parallel}\omega_{\parallel}(\alpha_{\parallel}\sin\theta_{\parallel}-\beta_{\parallel}\cos\theta_{\parallel})\ , \tag{2.25}$$

$$\mathbf{u}_1=c(\alpha_{\perp 1}\sin\theta_{\perp 1}+\alpha_{\perp 2}\sin\theta_{\perp 2},-\beta_{\perp 1}\cos\theta_{\perp 1}-\beta_{\perp 2}\cos\theta_{\perp 2},\frac{\omega_\parallel^2}{\omega_p^2}\alpha_\parallel\sin\theta_\parallel-\frac{\omega_\parallel^2}{\omega_p^2}\cos\theta_\parallel)\ , \tag{2.26}$$

$$\mathbf{r}_1=c[(\alpha_{\perp 1}/\omega_{\perp 1})\cos\theta_{\perp 1}+(\alpha_{\perp 2}/\omega_{\perp 2})\cos\theta_{\perp 2},(\beta_{\perp 1}/\omega_{\perp 1})\sin\theta_{\perp 1}+(\beta_{\perp 2}/\omega_{\perp 2})\sin\theta_{\perp 2}\ ,(\omega_\parallel/\omega_p^2)\alpha_\parallel\cos\theta_\parallel+(\omega_\parallel/\omega_p^2)\beta_\parallel\sin\theta_\parallel]\ . \tag{2.27}$$

Assuming the frequency resonance condition

$$\omega_{\perp 1}-\omega_{\perp 2}=\omega_\parallel\ , \tag{2.28}$$

we solve Eqs. (2.1)–(2.6), correct up to the second order of small quantities, and find $\langle \mathbf{H}^{\rm in}\rangle$ using Eq. (1.2):

$$\langle H_x^{\rm in}\rangle=0,\quad \langle H_y^{\rm in}\rangle=0\ , \tag{2.29}$$

$$\langle H_z^{\rm in}\rangle=\frac{\pi N_0 e}{2c}\frac{\omega_p^2 n_\parallel c^3}{X_\parallel C_s}\delta_\parallel B_\perp\sin(\Delta k z-\gamma)\ , \tag{2.30}$$

where

$$n_\parallel=\frac{k_\parallel C_s}{\omega_\parallel},\quad X_\parallel=\frac{\omega_p^2}{\omega^2}\ , \tag{2.31}$$

$$B_\perp=2(\alpha_{\perp 1}\beta_{\perp 2}+\alpha_{\perp 2}\beta_{\perp 1})\left[\frac{\delta_{2+}}{\omega_{\perp 1}}+\frac{\delta_{1-}}{\omega_{\perp 2}}\right]\ , \tag{2.32}$$

$$\delta_{1-}=\frac{1}{\omega_{\perp 2}^2-c^2(k_\parallel-k_{\perp 1})^2-\omega_p^2}\ ,\qquad \delta_{2+}=\frac{1}{\omega_{\perp 1}^2-c^2(k_\parallel+k_{\perp 2})^2-\omega_p^2}\ , \tag{2.33}$$

$$\delta_\parallel=(\alpha_\parallel^2+\beta_\parallel^2)^{1/2},\quad \gamma=\tan^{-1}\frac{\beta_\parallel}{\alpha_\parallel}=\tan^{-1}\frac{b_\parallel}{a_\parallel}\ , \tag{2.34}$$

$$\Delta k=k_\parallel-k_{\perp 1}+k_{\perp 2}$$
$$=\left[-\left[\frac{1}{c}\right](\omega_{\perp 1}^2-\omega_p^2)^{1/2}+\left[\frac{1}{c}\right](\omega_{\perp 2}^2-\omega_p^2)^{1/2}+\left[\frac{1}{C_s}\right](\omega_\parallel^2-\omega_p^2)^{1/2}\right]$$
$$\simeq\left[-\left[\frac{1}{c}\right](\omega_{\perp 1}-\omega_{\perp 2})+\frac{\omega_\parallel}{C_s}\right]\approx\frac{\omega_\parallel}{C_s}\ . \tag{2.35}$$

Δk is the wave number mismatch for the frequency-matching relation of (2.28). We simplify (2.33) with the help of (2.14) and (2.28), and obtain

$$\delta_{1-}\approx\delta_{2+}\approx-\frac{1}{\omega_p^2}\ . \tag{2.36}$$

Hence the formula of (2.30) effectively simplifies to

$$\langle H_z^{\rm in}\rangle=-\frac{\pi N_0 e n c^2}{C_s X_\parallel}\delta_\parallel\left[\frac{1}{\omega_{\perp 1}}+\frac{1}{\omega_{\perp 2}}\right](\alpha_{\perp 1}\beta_{\perp 2}+\alpha_{\perp 2}\beta_{\perp 1})\times\sin\left[\frac{\omega_\parallel}{C_s}-\gamma\right]\ . \tag{2.37}$$

Since this magnetization exists only along the common direction of propagation of the three waves, its spatial sinusoidal variation has only a longitudinal gradient. Distance between two successive layers of a maximum field strength is proportional to C_s/ω_p, taking $\omega_\parallel\approx\omega_p$, in plasmas.

C. The case of two electron acoustic waves and one transverse wave

In this case [subcase (b)]

$$\mathbf{E}_1=(a_\perp\cos\theta_\perp,b_\perp\sin\theta_\perp,(a_{\parallel 1}^2+b_{\parallel 1}^2)^{1/2}\cos(\theta_{\parallel 1}-\gamma_1)+(a_{\parallel 2}^2+b_{\parallel 2}^2)^{1/2}\cos(\theta_{\parallel 2}-\gamma_2))\ , \tag{2.38}$$

$$\mathbf{H}_1=\frac{mc^2}{e}(-\beta_\perp k_\perp\sin\theta_\perp,\alpha_\perp k_\perp\cos\theta_\perp,0)\ , \tag{2.39}$$

$$N_1=\frac{N_0 n_\parallel c}{X_{\parallel 1}C_s}\delta_{\parallel 1}\sin(\theta_{\parallel 1}-\gamma_1)+\frac{N_0 n_{\parallel 2}c}{X_{\parallel 2}C_s}\delta_{\parallel 2}\sin(\theta_{\parallel 2}-\gamma_2)\ , \tag{2.40}$$

$$\mathbf{u}_1=c\left[\alpha_\perp\sin\theta_\perp,-\beta_\perp\cos\theta_\perp,\left[\frac{\delta_{\parallel 1}}{X_{\parallel 1}}\right]\sin(\theta_{\parallel 1}-\gamma_1)+\left[\frac{\delta_{\parallel 2}}{X_{\parallel 2}}\right]\sin(\theta_{\parallel 2}-\gamma_2)\right]\ , \tag{2.41}$$

$$\mathbf{r}_1=c\left[\left[\frac{\alpha_\perp}{\omega_\perp}\right]\cos\theta_\perp,\left[\frac{\beta_\perp}{\omega_\perp}\right]\sin\theta_\perp,\left[\frac{\delta_{\parallel 1}}{X_{\parallel 1}\omega_{\parallel 1}}\right]\cos(\theta_{\parallel 1}-\gamma_1)+\left[\frac{\delta_{\parallel 2}}{X_{\parallel 2}\omega_{\parallel 2}}\right]\cos(\theta_{\parallel 2}-\gamma_2)\right]\ . \tag{2.42}$$

Solving Eqs. (2.1)–(2.6), up to the second order, when

$$\omega_{\parallel 1}+\omega_{\parallel 2}=\omega_{\perp} \tag{2.43}$$

we find the components of $\langle \mathbf{H}^{\rm in}\rangle$ from (1.2),

$$\langle H_X^{\rm in}\rangle=-\left[\frac{\pi N_0 e}{2c}\right]\frac{\delta_{\parallel 1}\delta_{\parallel 2}c^3\beta_{\perp}D}{X_{\parallel 1}X_{\parallel 2}}\cos(\Delta kz-\gamma_1-\gamma_2)\ , \tag{2.44}$$

$$\langle H_y^{\rm in}\rangle=-\left[\frac{\pi N_0 e}{2c}\right]\frac{\delta_{\parallel 1}\delta_{\parallel 2}C^3\alpha_{\perp}D}{X_{\parallel 1}X_{\parallel 2}}\cos(\Delta kz-\gamma_1-\gamma_2)\ , \tag{2.45}$$

$$\langle H_z^{\rm in}\rangle=0\ , \tag{2.46}$$

with

$$D=2(K_{\parallel 1}+K_{\parallel 2})\delta+\frac{2\omega_p^2}{C_s}\left[\frac{n_{\parallel 1}}{\omega_{\parallel 2}}\delta_{1-}+\frac{n_{\parallel 2}}{\omega_{\parallel 1}}\delta_{2-}\right]\ , \tag{2.47}$$

$$\delta=\frac{1}{\omega_{\perp}^2-C_s^2(k_{\parallel 1}+k_{\parallel 2})^2-\omega_p^2}\ , \tag{2.48}$$

$$n_{\parallel i}=\frac{k_{\parallel i}C_s}{\omega_{\parallel i}},\quad X_{\parallel i}=\frac{\omega_p^2}{\omega_{\parallel i}^2}\ , \tag{2.49}$$

$$\delta_{1-}=\frac{1}{\omega_{\parallel 2}^2-c^2(k_{\perp}-k_{\parallel 1})^2-\omega_p^2}\ , \tag{2.50}$$

$$\delta_{2-}=\frac{1}{\omega_{\parallel 1}^2-c^2(k_{\perp}-k_{\parallel 2})^2-\omega_p^2}\ . \tag{2.51}$$

Relations (2.14) and (2.43) give

$$\delta\approx\delta_{1-}\approx\delta_{2-}\approx-\frac{1}{\omega_p^2}\ , \tag{2.52}$$

$$\Delta k=k_{\parallel 1}+k_{\parallel 2}-k_{\perp}\approx\frac{\omega_{\perp}}{C_s}\ . \tag{2.53}$$

With the help of (2.52) and (2.53) formulas (2.44) and (2.45) are much simplified. Equations (2.44)–(2.46) show that this field is purely lateral.

D. The case of three transverse waves

In this case [subcase (c)]

$$\mathbf{E}_1=(a_{\perp 1}\cos\theta_{\perp 1}+a_{\perp 2}\cos\theta_{\perp 2}+a_{\perp 3}\cos\theta_{\perp 3},b_{\perp 1}\sin\theta_{\perp 1}+b_{\perp 2}\sin\theta_{\perp 2}+b_{\perp 3}\sin\theta_{\perp 3},0)\ . \tag{2.54}$$

Solutions of Eqs. (2.1)–(2.6), up to the first order, are

$$\mathbf{H}_1=\frac{mc^2}{e}(-\beta_{\perp 1}k_{\perp 1}\sin\theta_{\perp 1}-\beta_{\perp 2}k_{\perp 2}\sin\theta_{\perp 2}-\beta_{\perp 3}k_{\perp 3}\sin\theta_{\perp 3}\ ,\alpha_{\perp 1}k_{\perp 1}\cos\theta_{\perp 1}+\alpha_{\perp 2}k_{\perp 2}\cos\theta_{\perp 2}+\alpha_{\perp 3}k_{\perp 3}\cos\theta_{\perp 3},0)\ , \tag{2.55}$$

$$N_1=0\ , \tag{2.56}$$

$$\mathbf{u}_1=c(\alpha_{\perp 1}\sin\theta_{\perp 1}+\alpha_{\perp 2}\sin\theta_{\perp 2}+\alpha_{\perp 3}\sin\theta_{\perp 3},-\beta_{\perp 1}\cos\theta_{\perp 1}-\beta_{\perp 2}\cos\theta_{\perp 2}-\beta_{\perp 3}\cos\theta_{\perp 3},0)\ , \tag{2.57}$$

$$\mathbf{r}_1=c\left[\left[\frac{\alpha_{\perp 1}}{\omega_{\perp 1}}\right]\cos\theta_{\perp 1}+\left[\frac{\alpha_{\perp 2}}{\omega_{\perp 2}}\right]\cos\theta_{\perp 2}+\left[\frac{\alpha_{\perp 3}}{\omega_{\perp 3}}\right]\cos\theta_{\perp 3}\ ,\left[\frac{\beta_{\perp 1}}{\omega_{\perp 1}}\right]\sin\theta_{\perp 1}+\left[\frac{\beta_{\perp 2}}{\omega_{\perp 2}}\right]\sin\theta_{\perp 2}+\left[\frac{\beta_{\perp 3}}{\omega_{\perp 3}}\right]\sin\theta_{\perp 3},0\right]\ . \tag{2.58}$$

Dropping the subscript $\perp$, which is unnecessary now, we write

$$\omega_3=\omega_1+\omega_2 \tag{2.59}$$

and solve Eqs. (2.1)–(2.6) correct up to the second order; we find that

$$\langle H_x^{\rm in}\rangle=\pi N_0 ec^2(\beta_1 B_{23}^-+\beta_2 B_{13}^-+\beta_3 B_{12}^-)\sin(\Delta kz)\ , \tag{2.60}$$

$$\langle H_y^{\rm in}\rangle=-\pi N_0 ec^2(\alpha_1 B_{23}^-+\alpha_2 B_{13}^-+\alpha_3 B_{12}^+)\cos(\Delta kz)\ , \tag{2.61}$$

$$\langle H_z^{\rm in}\rangle=0\ , \tag{2.62}$$

where

$$B_{ij}^{\pm}=\frac{(\alpha_i\alpha_j\mp\beta_i\beta_j)(k_i\pm k_j)}{C_s^2(k_i\pm k_j)^2-(\omega_i\pm\omega_j)^2+\omega_p^2}\ , \tag{2.63}$$

$$\Delta k\approx-\frac{\omega_p^2}{2c}\left[\frac{1}{\omega_1}+\frac{1}{\omega_2}-\frac{1}{\omega_3}\right]\ . \tag{2.64}$$

Hence, in subcase (c), as in (b), only the lateral magnetization is generated. This field causes anomalous diffusion of plasma in the presence of wave fields. The sinusoidal spatial variation of the magnetization, perpendicular to its direction, is a transverse gradient, consequences of which are a drift and the related current of charges.

E. Some remarks

Subcase (a) is identified with the SRS instability when $N_e>N_c/4$, N_e being the electron density and N_c its critical value. In this case a photon of lower energy and a plasmon are scattered. Subcase (b) is identified with the two plasmon decay instability at $N_e=N_c/4$, in appropriate density ranges. Subcase (c) is a Compton scattering of two photons of low energies by a photon of high energy. Since the frequency of the acoustic wave is smaller than the frequency of the light waves, relatively more energy is transferred to the scattered light wave in subcase (a), as may be seen from the Manley-Rowe relations, or otherwise also. Specifically, the incident and scattered light waves interact to drive, via the ponderomotive force, an electrostatic beat wave with phase velocity

$v_{\rm ph}=(\omega_{\perp 1}-\omega_{\perp 2})/(k_{\perp 1}-k_{\perp 2})$ [4]. If $v_{\rm ph}>v_{\rm th}$ ($v_{\rm th}$ is the electron thermal velocity), then the beat wave (an electron plasma wave of SRS) is resonantly enhanced by plasma. If $v_{\rm ph}\sim v_{\rm th}$, then stimulated Compton scattering results; since the frequency of its scattered field is larger than that for Raman scattering, relatively more energy is thus scattered.

III. SBS OF AN ALFVEN WAVE IN FINITELY CONDUCTING MEDIA

A parametrically temporally exponentially growing magnetization (case II of Sec. I) obtains in a finitely electrically conducting MHD fluid, when the parametric growth rate of the system exceeds the rate of damping, which is possible if the pump power exceeds a certain value. This threshold value, for instability to occur, is different for supersonic, subsonic, and sonic Alfven waves. For subsonic or sonic pump Alfven waves, the instability occurs when the two signal waves are copropagating. But, for supersonic Alfven waves, the instability may or may not occur. For counterpropagating signal waves the instability condition is just the opposite.

A resonant amplification occurs if, in addition to the frequency-matching condition (1.8), the real part of the parametric shift of frequency equals the frequency of the signal Alfven wave, Eq. (1.9). Since the amplification frequency is proportional to the pump amplitude, for a large-amplitude pump Alfven wave this resonance is not possible; but this amplification is possible for a weak nonlinearity, for which the wave modes retain their basic identity, but are no longer completely independent. The quasilinear relaxation of an initially unstable particle distribution, when influenced by developing oscillations, can stabilize the growth at best during a time inversely proportional to the wave energy. This means that the neglect of nonlinear wave interaction cannot be justified, even for small amplitudes; its importance is evident from comparison of the quasilinear relaxation time with the time of energy distribution of different modes.

The light wave creates pressure through electrostriction; the resultant density change affects the susceptibility. Thus in SBS light pumps the sound wave which scatters it; the scattering creates a second, frequency-shifted, idler light wave of a parametric amplifier when the frequency- and wave-number-matching conditions are satisfied for conversion of the incident light wave to the acoustic wave and the scattered light wave [9]. For acoustical phonons, with frequency below 10^{10} cps, these matching conditions can always be satisfied [10]. The SBS of Alfven waves by sound waves in an isothermal, homogeneous plasma may be associated with the heat balance of sunspots due to the effective transfer of a significant fraction of the flux of energy of Alfven waves to the short waves by convective motion above the sunspot [11].

A. The basic field equations

All the three waves are diffusive due to finiteness of the electrical conductivity. For their complex frequencies we write

$$\bar{\omega}_l=\omega_l+i\gamma_l, \quad l=1,2,3 . \tag{3.1}$$

Here ω_1, ω_2, and ω_3 are the real frequencies and γ_1, γ_2, and γ_3 are the decay constants of the three waves. The basic field equations are

$$\dot{\mathbf{B}}+(\mathbf{v}\cdot\nabla)\mathbf{B}-(\mathbf{B}\cdot\nabla)\mathbf{v}+\mathbf{B}\,\mathrm{div}\mathbf{v}=\eta\nabla^2\mathbf{B} , \tag{3.2}$$

$$\frac{\partial}{\partial t}(\rho\mathbf{v})+(\rho\mathbf{v}v_j)_{,j}=-\nabla\left[p+\frac{H^2}{8\pi}\right]+\frac{1}{4\pi}(\mathbf{H}\cdot\nabla)\mathbf{H} , \tag{3.3}$$

$$\dot{\rho}+\mathrm{div}(\rho\mathbf{v})=0 , \tag{3.4}$$

where $\nabla\equiv(0,0,\partial/\partial z)$, $\eta(=c^2/4\pi\sigma)$ is the diffusivity of the medium, σ is the electrical conductivity, and the other symbols have their usual meanings. For paramagnetic materials, setting the magnetic permeability $\mu=1$, the components of the basic equations, in the direction of wave propagation, are

$$\rho\dot{v}_z+\rho v_z\frac{\partial v_z}{\partial z}=-C_s^2\frac{\partial\rho}{\partial z}-\frac{\partial}{\partial z}\left[\frac{H^2}{8\pi}\right] , \tag{3.5}$$

$$\dot{\rho}+\frac{\partial}{\partial z}(\rho v_z)=0 . \tag{3.6}$$

The components, in the perpendicular directions, are

$$\rho\dot{\mathbf{v}}_\perp+\rho v_z\frac{\partial\mathbf{v}_\perp}{\partial z}=\frac{H_z}{4\pi}\frac{\partial\mathbf{H}}{\partial z} , \tag{3.7}$$

$$\dot{\mathbf{H}}-\eta\frac{\partial^2\mathbf{H}}{\partial z^2}=H_0\frac{\partial\mathbf{v}_\perp}{\partial z}-\frac{\partial}{\partial z}(v_z\mathbf{H}) . \tag{3.8}$$

For the magnetic field, fluid velocity, and density of the medium, in the perturbed state, we write

$$\mathbf{H}=H_0\hat{\mathbf{z}}+\mathbf{H}_2+\mathbf{H}_3, \quad \mathbf{v}=v_1\hat{\mathbf{z}}+\mathbf{v}_2+\mathbf{v}_3, \quad \rho=\rho_0+\rho_1 . \tag{3.9}$$

ρ_0 is the unperturbed density of the medium, H_0 is the uniform background magnetic field acting along the z direction, ρ_1 is the first-order perturbation of the fluid density, and $\mathbf{v}_1$ is the velocity induced by the sound wave. The quantities $\mathbf{v}_2$ and $\mathbf{H}_2$ represent the first-order velocity and magnetic field of the signal Alfven wave and $\mathbf{v}_3$, $\mathbf{H}_3$ represent the same quantities for the pump Alfven wave.

B. The basic equations for parametric interaction

For study of the parametric interaction of the three waves the relevant portions of these equations are

$$\dot{v}_1+\frac{C_s^2}{\rho_0}\frac{\partial\rho_1}{\partial z}=-\frac{1}{4\pi\rho_0}\frac{\partial}{\partial z}(\mathbf{H}_2\cdot\mathbf{H}_3) , \tag{3.10}$$

$$\dot{\rho}_1+\rho_0\frac{\partial v_1}{\partial z}=0 , \tag{3.11}$$

$$\dot{\mathbf{v}}_2-\frac{H_0}{4\pi\rho_0}\frac{\partial\mathbf{H}_2}{\partial z}=-v_1\frac{\partial\mathbf{v}_3}{\partial z}-\frac{1}{\rho_0}(\rho_1\dot{\mathbf{v}}_3) , \tag{3.12}$$

$$\dot{\mathbf{H}}_2-\eta\frac{\partial^2\mathbf{H}_2}{\partial z^2}-H_0\frac{\partial\mathbf{v}_2}{\partial z}=-\frac{\partial}{\partial z}(v_1\mathbf{H}_3) , \tag{3.13}$$

$$\dot{\mathbf{v}}_3-\frac{H_0}{4\pi\rho_0}\frac{\partial \mathbf{H}_3}{\partial z}=0\ , \tag{3.14}$$

$$\dot{\mathbf{H}}_3-\eta\frac{\partial^2\mathbf{H}_3}{\partial z^2}-H_0\frac{\partial \mathbf{v}_3}{\partial z}=0\ . \tag{3.15}$$

These give the following inhomogeneous wave equations:

$$L_1^2 v_1=-\frac{1}{4\pi\rho_0}\frac{\partial^2}{\partial z\partial t}(\mathbf{H}_2\cdot\mathbf{H}_3)\ , \tag{3.16}$$

$$L_1^2\rho_1=\frac{1}{4\pi}\frac{\partial^2}{\partial z^2}(\mathbf{H}_2\cdot\mathbf{H}_3)\ , \tag{3.17}$$

$$L_2^3\mathbf{v}_2=\left[\frac{\partial}{\partial t}-\eta\frac{\partial^2}{\partial z^2}\right]\left[v_1\frac{\partial \mathbf{v}_3}{\partial z}\right]+\frac{1}{\rho_0}\left[\frac{\partial}{\partial t}-\eta\frac{\partial^2}{\partial z^2}\right](\rho_1\dot{\mathbf{v}}_3)+\left[\frac{C_A^2}{H_0}\right]\frac{\partial^2}{\partial z^2}(v_1\mathbf{H}_3)\ , \tag{3.18}$$

$$L_2^3\mathbf{H}_2=H_0\frac{\partial}{\partial z}\left[v_1\frac{\partial \mathbf{v}_3}{\partial z}\right]+\left[\frac{H_0}{\rho_0}\right]\frac{\partial}{\partial z}(\rho_1\dot{\mathbf{v}}_3)+\frac{\partial^2}{\partial z\partial t}(v_1\mathbf{H}_3)\ , \tag{3.19}$$

$$L_2^3(\mathbf{v}_3,\mathbf{H}_3)=0\ , \tag{3.20}$$

where

$$L_1^2=\frac{\partial^2}{\partial t^2}-C_s^2\frac{\partial^2}{\partial z^2},\quad L_2^3=\eta\frac{\partial^3}{\partial t\partial z^2}-\frac{\partial^2}{\partial t^2}+C_A^2\frac{\partial^2}{\partial z^2}\ . \tag{3.21}$$

Equations (3.16) and (3.17), being those for the sound wave, do not contain any term involving η; so its dispersion relation is

$$k_1^2C_s^2\approx\omega_1^2\ , \tag{3.22}$$

where k_1,ω_1 are the wave number and the real part of the frequency, and C_s $[=(k_BT_e|M)^{1/2}]$ represents the sound velocity, T_e is the electron temperature, M is the ion mass, and k_B is the Boltzmann constant.

The first harmonic field variables are

$$\mathbf{H}_l=(H_{l1}\hat{\mathbf{x}}+iH_{l2}\hat{\mathbf{y}})e^{i\bar{\theta}_l}+\text{c.c.}\quad (l=2,3)\ , \tag{3.23}$$

$$v_1=(v_{11}+iv_{12})e^{i\bar{\theta}_1}+\text{c.c.}\ , \tag{3.24}$$

where $\bar{\theta}_l=k_lz-\bar{\omega}_lt$ and c.c. means the complex conjugate. In the linearized approximation the amplitudes of the wave fields of (3.23) and (3.24) are constants. And, since

$$e^{i\bar{\theta}}=e^{\gamma t}e^{i\theta}\ \text{and}\ e^{-i\bar{\theta}^*}=e^{\gamma t}e^{-i\theta}\ , \tag{3.25}$$

we find that

$$\mathbf{H}_l=e^{\gamma_lt}[(H_{l1}\hat{\mathbf{x}}+iH_{l2}\hat{\mathbf{y}})e^{i\theta_l}+\text{c.c.}]\quad (l=2,3)\ ,$$

$$v_1=e^{\gamma_1t}[(v_{11}+iv_{12})e^{i\theta_1}+\text{c.c.}]\ , \tag{3.26}$$

where

$$\theta_l=k_lz-\omega_lt\quad (l=1,2,3)\ . \tag{3.27}$$

Using the solution of $\mathbf{H}_l$ $(l=2,3)$ from (3.23) in (3.20) we obtain for $\bar{\omega}$ the relation

$$i\eta\bar{\omega}_lk_l^2+\bar{\omega}_l^2-k_l^2C_A^2=0\ . \tag{3.28}$$

A similar relation is valid for its complex conjugate. Hence, equating real and imaginary parts from both sides of this relation and neglecting γ_l^2, we find that

$$k_l^2C_A^2\approx\omega_l^2\ \text{and}\ \gamma_l\approx-\eta k_l^2/2\ . \tag{3.29}$$

These are, respectively, the dispersion relations (in the linearized approximation) and the value of the decay constants for the signal and pump Alfven waves. Eventually, γ_2 and γ_3 are negative quantities, showing damping of the waves due to the finite electrical conductivity.

Equations (3.22), (3.29), and (3.20) give the wave-number-matching condition $k_3=k_1+k_2$, and the wave phase matching condition $\theta_3=\theta_1+\theta_2$. Hence, in a dissipative medium, the total momentum of the system remains conserved, though the energy conservation does not necessarily hold good owing to wave decay.

C. The relevant field variables and the wave equations

Substituting the wave solution for $\mathbf{H}_2$, $\mathbf{H}_3$, and v_1 in the linearized equations (3.11), (3.12), and (3.14), we obtain

$$\rho_1=\frac{\rho_0k_1}{\omega_1^2}e^{\gamma_1t}[(\omega_1-i\gamma_1)(v_{11}+iv_{12})e^{i\theta_1}+\text{c.c.}]\ , \tag{3.30}$$

$$\mathbf{v}_l=-\frac{H_0k_le^{\gamma_lt}}{4\pi\rho_0\omega_l^2}[(\omega_l-i\gamma_l)(\mathbf{H}_{l1}\hat{\mathbf{x}}+iH_{l2}\hat{\mathbf{y}})e^{i\theta_1}+\text{c.c.}]\quad (l=2,3) \tag{3.31}$$

Now we retain the first-order time derivatives of amplitudes of the idler waves. Since the pump wave is releasing its energy to these waves, it is not parametrically amplified, so its amplitude does not vary for such purposes. Thus proceeding, we obtain the following equations for evolution of the parametrically amplified signal wave:

$$D(v_{11}+iv_{12})=\frac{k_1}{8\pi\rho_0\omega_1^2}e^{(\gamma_2+\gamma_3-\gamma_1)t}(\omega_1-i\gamma_1)(\gamma_2+\gamma_3-i\omega_1)(H_{21}^*H_{31}+H_{22}^*H_{32})\ , \tag{3.32}$$

$$D(H_{21}\hat{\mathbf{x}}+iH_{22}\hat{\mathbf{y}})=-\frac{ik_2}{2\omega_2}e^{(\gamma_1+\gamma_3-\gamma_2)t}\left[(\omega_2+\omega_3+i\gamma_1)-\frac{k_1k_3C_A^2}{\omega_1^2}(\omega_1+i\gamma_1)\right](v_{11}^*-iv_{12}^*)(H_{31}\hat{\mathbf{x}}+iH_{32}\hat{\mathbf{y}})\ , \tag{3.33}$$

where $D \equiv \partial/\partial t$. Nontrivial solutions obtain for $\gamma_1=0$, because the sound wave is not affected by the conductivity of the medium. Hence

$$D(v_{11}+iv_{12})=\frac{k_1}{8\pi\rho_0\omega_1}e^{(\gamma_2+\gamma_3)t}(\gamma_2+\gamma_3-i\omega_1)\times(H_{21}^*H_{31}+H_{22}^*H_{32}) , \quad (3.34)$$

$$D(H_{21}\hat{\mathbf{x}}+iH_{22}\hat{\mathbf{y}})=-\frac{ik_2}{2\omega_2}e^{(\gamma_3-\gamma_2)t}\left[\omega_2+\omega_3\left[1-\frac{C_A}{C_s}\right]\right]\times(v_{11}^*-iv_{12}^*)(H_{31}\hat{\mathbf{x}}+iH_{32}\hat{\mathbf{y}}) . \quad (3.35)$$

Equation (3.34) gives

$$D[e^{-(\gamma_2+\gamma_3)t}D(v_{11}+iv_{12})] =\frac{k_1}{8\pi\rho_0\omega_1}(\gamma_2+\gamma_3-i\omega_1)D(H_{21}^*H_{31}+H_{22}^*H_{32}) . \quad (3.36)$$

Since the left side of this equation can be written as

$$[De^{-(\gamma_2+\gamma_3)t}D(v_{11}+iv_{12})] +[e^{-(\gamma_2+\gamma_3)t}D^2(v_{11}+iv_{12})] , \quad (3.37)$$

the assumed solutions are

$$(v_{11}+iv_{12})=(\hat{v}_{11}+i\hat{v}_{12})e^{i\varpi t} , \quad (3.38)$$

$$(H_{21}\hat{\mathbf{x}}+iH_{22}\hat{\mathbf{y}})=(\hat{H}_{21}\hat{\mathbf{x}}+i\hat{H}_{22}\hat{\mathbf{y}})e^{i\varpi t} , \quad (3.39)$$

where $\varpi=\omega+i\gamma$; ω is the real, parametrically evolved, frequency shift and γ is the growth rate (also called the amplification constant); and $D\equiv\partial/\partial t=i\varpi=i\omega-\gamma$. Since γ_2 and γ_3 are small, compared to γ, for occurrence of the parametric instability, the first term in (3.37) can be neglected compared to the second one; then (3.36) becomes

$$D^2(v_{11}+iv_{12})=\frac{ik_1k_2e^{2\gamma_3 t}}{16\pi\rho_0\omega_1\omega_2}(\gamma_2+\gamma_3-i\omega_1)\times\left[\omega_2+\omega_3\left[1-\frac{C_A}{C_s}\right]\right]\times(v_{11}+iv_{12})|H_3|^2 . \quad (3.40)$$

Now, substituting the value of $v_{11}+iv_{12}$ from (3.38) in this relation we obtain

$$\varpi^2=-\frac{k_1k_2e^{2\gamma_3 t}}{16\pi\rho_0\omega_1\omega_2}[\omega_1+i(\gamma_2+\gamma_3)]\times\left[\omega_2+\omega_3\left[1-\frac{C_A}{C_s}\right]\right]|H_3|^2 . \quad (3.41)$$

So, both ω and γ depend upon the pump energy ($=H_3^2/8\pi$) which is initially very large; γ_2 and γ_3 are, therefore, small compared to γ. When the two signal waves are propagating in the same direction (that is, if $k_1k_2>0$), the following three cases of evolution of the Alfven wave arise:

$$\text{(i) } C_A<C_s \text{ (subsonic)}, \quad \text{(ii) } C_A=C_s \text{ (sonic)} , \quad \text{(iii) } C_A>C_s \text{ (supersonic)} . \quad (3.42)$$

For $C_A<C_s$, since $\omega_2+\omega_3(1-C_A/C_s)>0$, we find that

$$\omega+i\gamma=\pm i(k_1k_2/16\pi\rho_0\omega_1\omega_2)^{1/2}e^{\gamma_3 t}\times[\omega_1+i(\gamma_2+\gamma_3)]^{1/2}\times\left[\omega_2+\omega_3\left[1-\frac{C_A}{C_s}\right]\right]^{1/2}|H_3| , \quad (3.43)$$

where

$$[\omega_1+i(\gamma_2+\gamma_3)]^{1/2}\approx\omega_1^{1/2}\left[\cos\frac{\theta}{2}+i\sin\frac{\theta}{2}\right] , \quad (3.44)$$

$$\theta=\tan^{-1}\left[\frac{\gamma_2+\gamma_3}{\omega_1}\right] , \quad (3.45)$$

which can also be expressed as

$$\theta=\tan^{-1}(B\gamma_3) , \quad (3.46)$$

with

$$B=\frac{1}{\omega_1}\left[2-2\frac{\omega_1C_A}{\omega_3C_s}+\frac{\omega_1^2C_A^2}{\omega_3^2C_s^2}\right] . \quad (3.47)$$

Here the squares and product terms of γ_2 and γ_3 have been neglected. Now, equating the real and imaginary parts from both sides of (3.43) we get

$$\omega=\mp Ae^{\gamma_3 t}\sin\frac{\theta}{2}, \quad \gamma=\pm Ae^{\gamma_3 t}\cos\frac{\theta}{2} , \quad (3.48)$$

where

$$A=\left[\frac{k_1k_2}{16\pi\rho_0\omega_2}\right]^{1/2}\left|\omega_2+\omega_3\left[1-\frac{C_A}{C_s}\right]\right|^{1/2}|H_3| . \quad (3.49)$$

Since $\gamma_3\approx O(1/\sigma)$, we have $t=1/\gamma_3\approx O(\sigma)$, so it will take a long time for the pump wave to decay at the rate of γ_3. Hence, at $t=0$, we obtain

$$\omega=\mp A\sin\frac{\theta}{2}, \quad \gamma=\pm A\cos\frac{\theta}{2} . \quad (3.50)$$

The threshold limit of the parametric instability is determined by setting $\gamma=\gamma_2$ which means having $\gamma_2=A\cos\theta/2$. Moreover, this relation, together with $\gamma_2=-\eta k_2^2/2$, yields

$$W_H=\frac{H_3^2}{8\pi}=\frac{\omega_2^4C^4\rho_0^3C_AC_s}{2H_0^4\sigma^2\omega_1}\sec^2\frac{\theta}{2}\left[\omega_2+\omega_3\left[1-\frac{C_A}{C_s}\right]\right]^{-1} \quad (3.51)$$

for the threshold pump energy for the onset of such instability.

Similarly, for a sonic Alfven wave,

$$\gamma=\gamma_2=\pm\left[\frac{k_1k_2}{16\pi\rho_0}\right]^{1/2}|H_3|\cos\frac{\theta}{2}$$

and

$$W_H=\frac{\omega_2^4c^4\rho_0^3C_AC_s}{2H_0^4\sigma^2\omega_1\omega_2}\sec^2\frac{\theta}{2} \tag{3.52}$$

for the threshold pump energy. Since $\omega_3>\omega_2$, the frequency-matching condition $\omega_3=\omega_1+\omega_2$, for a supersonic Alfven wave, gives $\omega_2+\omega_3(1-C_A/C_s)<0$. Hence

$$\omega+i\gamma=\pm\left[\frac{k_1k_2}{16\pi\rho_0\omega_1\omega_2}\right]^{1/2}e^{\gamma_3 t}[\omega_1+i(\gamma_2+\gamma_3)]\times\left[\omega_2+\omega_3\left[1-\frac{C_A}{C_s}\right]\right]^{1/2}|\mathbf{H}_3| , \tag{3.53}$$

and consequently,

$$\omega=\pm A\cos\frac{\theta}{2}\quad\text{and}\quad\gamma=\pm A\sin\frac{\theta}{2}\quad\text{at } t=0 . \tag{3.54}$$

Since γ_2 and γ_3 are small, from (3.43) we find that θ is either very close to zero or is any multiple of π. Thus, for a supersonic Alfven wave, parametric instability may or may not occur. The reverse situation occurs for all the three cases of subsonic, sonic, and supersonic Alfven waves, if the two signal waves propagate in opposite directions (i.e., for $k_1k_2<0$).

The electric field components of the pump wave,

$$E_{3x}=\frac{ie^{\gamma_3 t}}{k_3c}[(\omega_3+i\gamma_3)H_{32}e^{i\theta_3}+\text{c.c.}] , \tag{3.55}$$

$$E_{3y}=-\frac{e^{\gamma_3 t}}{k_3c}[(\omega_3+i\gamma_3)H_{31}e^{i\theta_3}+\text{c.c.}] , \tag{3.56}$$

are obtained if $\mathbf{H}_3$ is substituted from (3.26) in the Maxwell equation (2.3). Then, the pump field Poynting vector,

$$\frac{c}{4\pi}(\mathbf{E}_3\times\mathbf{H}_3)=\left[0,0,\frac{C_A}{2\pi}|H_3|^2e^{2\gamma_3 t}\right] , \tag{3.57}$$

is exclusively along the direction of the field propagation. The initial value,

$$\frac{C_A}{2\pi}|H_3|^2 , \tag{3.58}$$

decays exponentially with time, because $\gamma_3<0$.

IV. STIMULATED EXCITATION OF A MOMENT FIELD IN SBS OF ALFVEN WAVES

The fluid displacement ξ is given by

$$\mathbf{v}=\frac{d\xi}{dt}=\frac{\partial\xi}{\partial t}+(\mathbf{v}\cdot\nabla)\xi . \tag{4.1}$$

For the signal Alfven wave, neglecting the higher-order convective term, we have

$$\mathbf{v}_2=\frac{\partial\xi_2}{\partial t} . \tag{4.2}$$

Substituting $\mathbf{v}_2$ from (3.31) in the relation (4.2) and then integrating with respect to time, we get

$$\xi_2=-\frac{iH_0k_2e^{(\gamma_2-\gamma)t}}{4\pi\rho_0\omega_2^2\{(\omega_2-\omega)^2+(\gamma_2-\gamma)^2\}}\times[(\omega_2-i\gamma_2)\{(\omega_2-\omega)-i(\gamma_2-\gamma)\}\times(\hat{H}_{21}\hat{\mathbf{x}}+iH_{22}\hat{\mathbf{y}})e^{i\{k_2z-(\omega_2-\omega)t\}}+\text{c.c.}] . \tag{4.3}$$

Substituting $\mathbf{H}_2$ from (3.23) in the MHD equation (1.7) and assuming

$$H_{21}\hat{\mathbf{x}}+iH_{22}\hat{\mathbf{y}}=(\hat{H}_{21}\hat{\mathbf{x}}+i\hat{H}_{22}\hat{\mathbf{y}})e^{i\bar{\omega}t} \tag{4.4}$$

we obtain

$$\mathbf{j}_2=\frac{ik_2c}{4\pi}e^{(\gamma_2-\gamma)t}\times[(\hat{H}_{21}\hat{\mathbf{y}}-i\hat{H}_{22}\hat{\mathbf{x}})e^{i\{k_2z-(\omega_2-\omega)t\}}+\text{c.c.}] . \tag{4.5}$$

The magnetic dipole moment, per unit volume, due to the signal Alfven wave, is then evaluated from formula (1.6).

A. Magnetization in a finitely conducting fluid

For finitely conducting fluids, we find that

$$\boldsymbol{\mu}=-\frac{e^{2(\gamma_2-\gamma)t}}{8\pi H_0\{(\omega_2-\omega)^2+(\gamma_2-\gamma)^2\}}\times[(\omega_2-i\gamma_2)\{(\omega_2-\omega)-i(\gamma_2-\gamma)\}+\text{c.c.}]|H_2|^2\hat{\mathbf{z}} . \tag{4.6}$$

Therefore the nonoscillating magnetic field is given by

$$\mathbf{H}^{\text{in}}=4\pi\boldsymbol{\mu}=-\frac{e^{2(\gamma_2-\gamma)t}}{2H_0\{(\omega_2-\omega)^2+(\gamma_2-\gamma)^2\}}\times[(\omega_2-i\gamma_2)\{(\omega_2-\omega)-i(\gamma_2-\gamma)\}+\text{c.c.}]|\hat{H}_2|^2\hat{\mathbf{z}} . \tag{4.7}$$

It is along the direction of wave propagation only. Eliminating $|\hat{H}_2|^2$ we obtain

$$H^{\text{in}}=-\frac{e^{2(\gamma_3-\gamma)t}[\omega_2+\omega_3(1-C_A/C_s)]^2}{8H_0C_A^2(\omega^2+\gamma^2)\{(\omega_2-\omega)^2+(\gamma_2-\gamma)^2\}}\times[(\omega_2-i\gamma_2)\{(\omega_2-\omega)-i(\gamma_2-\gamma)\}+\text{c.c.}]\times\hat{v}_1^2|H_3|^2 \tag{4.8}$$

for the z component of this field; the other two components vanish.

Instability occurs for negative values of γ. By (3.29), γ_3 is also negative. Hence, for $|\gamma|<|\gamma_3|$, the induced field decays exponentially. At $\gamma=\gamma_3$, it becomes constant, and for $|\gamma|>|\gamma_3|$, a temporally exponentially growing field is obtained. At the threshold point of growth of the parametric instability, the induced magnetic field is

$$H_i = -\frac{e^{2(\gamma_3-\gamma_2)t}[\omega_2+\omega_3(1-C_A/C_s)]^2\omega_2}{4H_0C_A^2(\omega^2+\gamma_2^2)(\omega_2-\omega)}\hat{v}_1^2|H_3|^2 \ . \tag{4.9}$$

B. SEMF from a double resonance

Comparing (4.8) and (4.14) we observe that, in a finitely conducting fluid, resonance is possible if $\omega=\omega_2$ in addition to $\omega_3=\omega_1+\omega_2$. However, this double resonance is not possible in an infinitely conducting fluid. Since γ_2 and γ_3 are both small in magnitude, the induced field grows slowly; and from (3.49), (3.50), and (3.54), the amplification frequency ω is found to depend directly upon the pump energy. So, for a strong pump Alfven wave this double resonance cannot occur if the signal Alfven wave frequency is very low. For a subsonic Alfven wave, at the double resonance due to $\omega=\omega_2$, the pump energy W_R is given by

$$W_R = \frac{2\rho_0 C_A C_s \omega_2^2}{\omega_1[\omega_2+\omega_3(1-C_A/C_s)]\sin^2\theta/2} \ . \tag{4.10}$$

This energy should be less than the threshold pump energy; so at the critical point of parametric instability, we have

$$\frac{W_H}{W_R} \approx \frac{c^4\omega_2^2\tan^2(\theta/2)}{64\pi^2\sigma^2C_A^4} \ . \tag{4.11}$$

For high electrical conductivity, $\theta\approx\pi/2$, so

$$W_H \gg W_R \ . \tag{4.12}$$

Since this SEMF from a double resonance is possible when the pump energy is not strong and the electrical conductivity is finite, after a long relaxation time, an instability will ultimately develop due to a weak nonlinearity. A similar situation also develops for weakly nonlinear supersonic and sonic pump Alfven waves.

C. Magnetization in an infinitely conducting fluid

In an infinitely conducting, compressible MHD fluid, the pump and signal Alfven waves propagate undamped in the same direction with real wave numbers k_3 and k_2 are real frequencies ω_3 and ω_2, respectively. They interact parametrically with a sound wave of frequency ω_1 and wave number k_1, such that

$$\omega_3=\omega_1+\omega_2, \quad k_3=k_1+k_2 \ . \tag{4.13}$$

Hence the total energy and momentum of the system remain conserved, and energy is effectively transferred from the pump Alfven wave to the two signal waves. The SEMF of this SBS process, in a conservative system, is due to amplification of the signal Alfven wave; the acoustic wave has no contribution to the magnetic moment because it has no current transverse to its direction of propagation.

When the electrical conductivity is infinite, we set $\gamma_2=0$, $\gamma_3=\gamma$ and, replacing ω by γ in (1.6), obtain

$$H_i = -\frac{e^{2\gamma t}[\omega_2+\omega_3(1-C_A/C_s)]^2}{4H_0C_A^2\gamma^2}\hat{v}_1^2|H_3|^2 \ , \tag{4.14}$$

where γ $[=\{(k_1^2\omega_3/16\omega_1)(H_3^2/4\pi\rho_0)\}]$ is the growth rate of the conservative system.

V. SOME REMARKS

A. General remarks

Three Faraday effects of electrodynamics are known to us to exist. One is the Faraday law of electromagnetic induction, which states that the rate of change of magnetic flux gives rise to the electromotive force of an electric field. The second Faraday effect is the Faraday rotation, which is the rotation of the plane of polarization of a plane-polarized electromagnetic wave, by an ambient magnetic field acting along its direction of propagation. The third one is the magnetization of the inverse Faraday effect (IFE). Originally, the IFE was conceived as the field from magnetic moments induced only by electric current of circularly polarized waves, in crystals and plasmas; so it was essentially the inverse of Faraday rotation effect [1–6,12]. However, this magnetic-moment field also exists for all bendings of motion of plasma constituents by waves. So, physically the effect is of more general occurrence than from the earlier conceived IFE, from waves of circular polarization only. It can be evaluated in other cases of wave-plasma interaction. Any search for a nonoscillating field, and particularly for a temporarily growing nonoscillating field, thus generated, seems important, because it controls the features of the relevant parametric instability.

We have here investigated theories of examples of two physical processes of generation of this magnetization, which have been designated as the SEMF and REMF, by us. One example per process has been considered for elucidation. Also we have initiated brief discussion on their application. We intend to begin further work on application soon.

In plasmas this effect was first calculated for nonrelativistic one-electron dipole approximation, considering the rotating electric field of a circularly polarized microwave radiation, which drives the electrons into circular orbits. Since the ions are much heavier than electrons, their effect was neglected to a first approximation [12]. Steiger and Woods [6] studied the same effect for interaction of plasmas with a laser beam. For strong, high-frequency fields the static magnetic field produced by the ion motion nearly cancels the field produced by the electron motion [13]. Such IFE fields occur in times which are shorter than twice the oscillation period of the driving field, so that, beyond such time scales, the wave becomes unstable and parametric instability develops [14,15]. This field has been investigated for interaction of propagating and standing Alfven waves with plasma [2].

Problems of plasma heating by lasers, employing two lasers, one of which is strong and the other weak, have been studied [16]. The resonant wave-plasma interaction of subcases (a) and (c), where the beat wave is a local noise field, may be used for understanding this heating.

A plasma interacts with coherent dispersive waves, which include local noise fields and strong driving fields, for rise or fall of the quasistatic magnetic field, both in small regions, and globally almost uniformly, for an entire body. Such growing fields generate bunching of plasma and energy evaporation for short durations, from strong synchrotron and bremsstrahlung radiations. For instance, puffs, or blobs of plasma are produced in laboratories, which have short life. These exhibit flares of still shorter durations, which may be due to such growing fields. Solar prominences and polar glows could be the consequences of this type of magnetization from parametric coupling of local noise fields with strong driving fields which emerge from disturbances elsewhere in outer space, and pass through the regions of the glows. Charges might become the elements of cosmic rays by getting a large amount of kinetic energy from such growing fields.

The resonance due to a matching of frequencies of first harmonics of three waves occurs in regions which are small compared to the largest of the wavelengths involved. In regions marked by large density gradients, the spatial distortions break the large-amplitude waves. It seems that the direction of the gradient of spatial inhomogeneities of the plasma is then important, because that decides the direction of the change of momentum and whether, as a consequence, the inhomogeneity can provide stabilization. The role of the magnetic-moment field in such processes is not clear to us.

The field from the thermal gradient source ($\nabla N \times \nabla T$) [17], rippled surface irregularities [18], and the dynamo effect [19], have been investigated. Contributions from these and other sources of field generation determine the direction and magnitude of the quasistatic magnetic field of a magnetized plasma. Obviously the diamagnetic sources reduce and paramagnetic sources increase the value of this field vector.

B. Remarks on application

The electrical conductivity is likely to be high in the latest stage of stellar evolution, where inside the star carbon and oxygen burn at temperatures of the order of 10^9 to 10^{10} K; the mass density $\rho_0 \approx 10^9$ to 10^{10} g/cm^3, $H_0 \approx 10^{10}$ G, the sound velocity C_s is about 10^9 cm/s. Then the phase velocity of Alfven waves is $C_A = 10^5$ cm/s (these are subsonic Alfven waves) and the ion gyration frequency $\Omega_i = 9.6 \times 10^{13}$ rad/s. A pump Alfven wave (wave 3) of electric field amplitude $E_3 = 10$ V/m ($= 10^{-3}/3$ esu/cm of electric field intensity) and frequency $\omega_3 = 2000$ rad/s is assumed to act on such a superdense plasma. Its magnetic component has the amplitude $H_3 (= c/2C_A) E_3 = 50$ G, which is evaluated from (2.3). This pump field interacts parametrically in a SEMF, with a signal sound wave (wave 1) and a signal Alfven wave (wave 2). We take $\omega_2 = 1500$ rad/s; then the frequency-matching condition (1.8) gives $\omega_1 = 500$ rad/s. The relations (3.22) and (3.29) yield $k_1 = 5 \times 10^{-7}$/cm, $k_2 = 1.5 \times 10^{-2}$/cm, $k_3 = 2 \times 10^{-2}$/cm. Assuming the electrical conductivity $\sigma = 10^{18}$/s, we find that the plasma diffusivity $\eta (= c^2/4\pi\sigma) = 72$ esu, $\gamma_2 (= -\eta k_2^2/2) = -8 \times 10^{-4}$/s, $\gamma_3 (= -\eta k_3^2/2) = -1.4 \times 10^{-2}$/s. The Poynting flux of the pump wave, evaluated from the relation (3.57), is 4×10^7 ergs/s/cm^2. The relations (3.48), for growth rate of the parametric instability, give $\gamma = 10^{-7}$/s. The threshold pump energy density, evaluated from (3.51), is $W_H = 1.2 \times 10^{16}$ ergs/cm^2.

Since this energy density is directly proportional to the cube of the plasma mass density, and inversely proportional to the fourth power of the ambient magnetic field, the threshold value is significant for very dense plasmas and even for moderate values of the ambient field. The temporally exponentially growing field of the parametric instability locally increases the distributions of fast particles along its direction and so enhances the tendency of anisotropy of the plasma there. The accelerating particles, in turn, generate Alfven waves [20]. Also, the very high pump wave energy, at the switch-on instant, soon becomes depleted as the signal waves begin to extract appreciable amounts of energy from it. If the signal sound wave is thus driven to a large amplitude, it may trap and accelerate particles in its potential trough, and thus be damped at a rate longer than the linear damping rate. For a large effective damping, the pump intensity may be below a threshold value and the instability may be switched off.

In the ionosphere the electron temperature is about 300 K, the number density ($N_i \approx N_0$) $\sim 10^6$ cm^{-3}, and the ambient magnetic field $H_0 \sim 0.3$ G; the corresponding ion gyration frequency is 3000 s^{-1}. We assume the frequencies of the large- and small-amplitude Alfven waves to be 375 and 250 s^{-1}, respectively, and their common phase velocity C_A to be 10^7 cm/s. Resonance occurs when a sound wave of frequency 125 s^{-1} propagates with sound velocity 1.86×10^5 cm/s. If the amplitude of the pump wave is 10 V/m, then $|H_3|^2 = 0.03$. The characteristic time is γ^{-1} ($= 9.2 \times 10^{-5}$ s), for growth of the field from the initial value 0.032 G to 0.224 G.

The brighter parts of the spectrum luminosity functions, now observed in the Sb, Sc, and Irr galaxies will remain in a steady state for at least 10^{10} years [21]. This slow evolution process may be a consequence of the weak nonlinearity, considered in Sec. IV B, which develops in the galactic systems. For stars with masses greater than the "white dwarf limit" a star cannot stabilize itself by attaining a degenerate configuration and, as it successively exhausts various sources of fuel, it will pass through higher and higher density and temperature ranges. It is now believed that at some point, strongly endothermic nuclear processes, rendering the star unstable on a very short time scale, will occur. An enormous amount of energy may be thus released in a short time scale, due to collapse of a star, which would be detected as a supernova explosion.

VI. CONCLUSIONS

Spontaneous magnetization is generated from quadratic nonlinearities for a matching of frequencies of first harmonic of three high-frequency waves (sum of frequencies of two of the waves equalling the frequency of the third wave), all propagating in the same direction. This magnetization may be called the resonant excitation of magnetic field, or REMF in short. It causes enhancement of the synchrotron radiation loss, and plasma bunching, which increases the collision frequency, and the consequent bremsstrahlung loss. Due to a wave-number mismatch, it has a sinusoidal spatial variation in the direction of propagation. In the case of two transverse waves and one electron acoustic wave, this magnetization is axial (that is, along the direction of wave propagation); so it has a longitudinal gradient. The gradient causes increase of plasma bunching in regions of maximum field strength. In the case of interaction of one transverse wave and two electron acoustic waves with a plasma, as well as in the case of interaction of three transverse waves, this magnetization exists only in a lateral direction, relative to the direction of propagation. It augments anomalous diffusion of plasma in the presence of wave fields. Evidently, it also has a sinusoidally varying transverse gradient, which causes a drift and a current of charges.

When a pump Alfven wave interacts with a signal Alfven wave and a signal sound wave, all propagating in the same direction, in a finitely electrically conducting MHD fluid, the parametric instability which develops when the pump power crosses a threshold value, also generates a temporally exponentially growing, nonoscillating magnetization, which may be called the stimulated excitation of the magnetic moment field, or SEMF, in short. All the various features which are associated with the growth of instability in a plasma, due to wave fields, are effectively controlled by this predicted magnetization. The threshold value of the pump power, for instability, is different for subsonic, sonic, and supersonic Alfven wave phase velocity. For subsonic, or sonic Alfven waves, the instability occurs when the two signal waves are copropagating. But, for supersonic Alfven waves, the instability may not occur. For counterpropagating signal waves the instability condition is just the opposite.

At the threshold limit of the pump power, a resonant amplification occurs when the real part of the parametric shift of the frequencies equals the frequency of the signal Alfven wave. The amplification frequency being proportional to pump amplitude, for a large-amplitude pump Alfven wave, this resonance is not possible; it is possible for a weak pump Alfven wave, because then the wave modes retain their identity though they are no longer completely independent.

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